Solutions to selected problems:

1. $\lim\limits_{x \to -5^-} \dfrac{3x^2 - x - 4}{x+5}$

$$\lim_{x \to 5^-} \frac{3x^2 - x - 4}{x+5} = \frac{3(-5)^2 - (-5) - 4}{(-5)+5} = \frac{76}{0},$$

which means the answer is either $\infty$ or $-\infty$.
We are approaching -5 from the left;
therefore, the values of $x$ are less than -5.
Therefore, $x+5$ is slightly negative. So,
our 'answer' of $\dfrac{76}{0}$ is really a positive
value divided by a 'slightly negative'
zero. So, the answer must be negative:
$\boxed{-\infty}$.

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2. Use the limit definition of a derivative to find the derivative of the function:

$$g(x) = \frac{3}{x}$$

$$g(x) = \frac{3}{x}, \quad g(x+h) = \frac{3}{x+h}$$

By the limit definition,

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{3}{x+h} - \frac{3}{x}\right) \bullet x(x+h)}{h \bullet x(x+h)}$$

$$= \lim_{h \to 0} \frac{3x - 3(x+h)}{hx(x+h)} = \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}x(x+h)}$$

$$= \lim_{h \to 0} \frac{-3}{x(x+h)} = -\frac{3}{x^2}$$

$$\boxed{g'(x) = -\frac{3}{x^2}}$$

* Test strategy hint: Verify that this answer is correct by using the Power Rule.

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3. A particle moves along the graph of $xy = x + 10$ so that $\frac{dx}{dt} = 4x + 4$.

What is $\frac{dy}{dt}$ when $x = 2$?

1. Background info / formula: $xy = x + 10$

2. Given info: $\frac{dx}{dt} = 4x + 4$

3. Find ( ), when ( ): Find $\frac{dy}{dt}$ when $x = 2$. We'll also need to know the value of $y$ when $x = 2$, so use the formula from part 1 to find it:

$xy = x + 10 \quad \to \quad 2y = 2 + 10 \quad \to \quad y = 6.$

So, really it's "Find $\frac{dy}{dt}$ when $x = 2$ and $y = 6$."

Now, differentiate the formula from part 1 with respect to $t$. Use the product rule for the first term:

$$x \bullet \frac{dy}{dt} + y \bullet \frac{dx}{dt} = \frac{dx}{dt}$$

Plug in the given info from part 2 and the "when" from part 3:

$$x \bullet \frac{dy}{dt} + y \bullet \frac{dx}{dt} = \frac{dx}{dt} \quad \to \quad x \bullet \frac{dy}{dt} + y \bullet (4x + 4) = 4x + 4$$

$$\to \quad 2 \bullet \frac{dy}{dt} + 6 \bullet (4 \bullet 2 + 4) = 4 \bullet 2 + 4$$

Solve for the "find":

$$\boxed{\frac{dy}{dt} = -30}$$

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